Warm-up!

A) I am thinking of a geometric sequence with g and g 45. Find g_{11}.

$g_8 = g_2 R^6$

$45 = 5R^6$

$9 = R^6$

$R = (9)^{1/6} = \sqrt[6]{9}$

$R = (3^2)^{1/6} = 3^{1/3} = \sqrt[3]{3}$

$g_{11} = g_8 R^3 = 45(\sqrt[6]{9})^3 = 45(9)^{\frac{3}{6}} = 45(9)^{1/2} = 45 \cdot 3$

$= 135$

B) I am thinking of an exponential function with 0. If $f(10) = 13$ and $f(12) = 26$, find).

$g_{10} = 13 \quad g_{12} = 26$

$g_{12} = g_{10} R^2$

$26 = 13R^2$

$2 = R^2$

$R = \sqrt{2}$

$g_{16} = g_{12} R^4$

$g_{16} = 26(\sqrt{2})^4 = 26(2)^{4/2}$

$26(2)^2$

$26 \cdot 4$

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C) Same exponential function as part B. Find

$g_{10.5} = g_{10}(R)^{0.5}$

$= 13(\sqrt{2})^{1/2}$

$= 13(2^{1/2})^{1/2}$

$= 13 \cdot 2^{1/4}$

$= 13\sqrt[4]{2}$

Exponential Functions Day 3:

- Exponent Properties
- Transforming and manipulating exponential functions
- Writing end behavior with words

Homework: yes

Quiz on Day 5:

- Essentially Day 1-3
- See Canvas for further details

Review: Exponent Properties

Product of Powers: $a^m \cdot a^n =$ a^{m+n}

Power of a Power: $(a^m)^n =$ $a^{m \cdot n}$

Power of a Product: $(ab)^m =$ $a^m b^m$

$\cancel{(a+b)^m \neq a^m + b^m}$

Negative Exponent: $a^{-m} =$ $\left(\frac{1}{a}\right)^m = \frac{1}{a^m}$

Zero Exponent: $a^0 =$ 1

Quotient of Powers: $\frac{a^m}{a^n} =$ a^{m-n}

Power of a Quotient: $\left(\frac{a}{b}\right)^m =$ $\frac{a^m}{b^m}$

Rational Exponents: $a^{\frac{m}{n}} =$ $\left(\sqrt[n]{a}\right)^m$

$= \sqrt[n]{a^m}$

Evaluate without the use of a calculator

A) $\frac{9^{-\frac{5}{4}}}{\left(\frac{1}{3}\right)^{\frac{3}{2}}} = \frac{\left(3^2\right)^{-5/4}}{\left(3^{-1}\right)^{3/2}} = \frac{3^{-10/4}}{3^{-3/2}} = \frac{3^{-5/2}}{3^{-3/2}}$

$= 3^{-\frac{5}{2} - \left(-\frac{3}{2}\right)}$

$= 3^{-2/2} = 3^{-1} = \frac{1}{3}$

B) $\left(8^{\frac{1}{3}} \cdot 8\right)^{\frac{1}{2}} = 8^{\frac{1}{6}} \cdot 8^{\frac{1}{2}} = 8^{2/3} = 4$

$\left(8^{\frac{1}{3}} \cdot 8^{1}\right)^{\frac{1}{2}} = \left(8^{4/3}\right)^{1/2} = 8^{4/6} = 8^{2/3}$

$= \left(\sqrt[3]{8}\right)^2$

$= 2^2$

$= 4$

Evaluate without the use of a calculator

C) $\frac{3^8 \cdot 3^4}{\left(9^{\frac{1}{4}}\right)^{20}}$

D) $2^{\frac{1}{2}} \cdot 18^{\frac{1}{2}}$

Let's just play with these and rewrite them in various ways using exponent properties $\frac{12}{8^x} = 12 \cdot \frac{1}{8^x} = 12 \cdot \left(\frac{1}{8}\right)^x$

$12 \cdot 2^{-3x}$

$$f(x) = \underbrace{12(2)^{-3x}}_{1} = \frac{12}{2^{3x}} = \frac{12}{(2^3)^x} = \frac{12}{8^x} = 12\left(\frac{1}{8}\right)^x$$

$12(8)^{-x}$

$6 \cdot 2^1 \cdot (2)^{-3x} = 6(2)^{1-3x} = 3 \cdot 2^1 \cdot 2^{1-3x} = 3 \cdot (2)^{2-3x}$

$$g(x) = 5\left(\frac{1}{3}\right)^{x-2} = 5\left(\frac{1}{3}\right)^x \cdot \left(\frac{1}{3}\right)^{-2}$$

$5\left(\frac{1}{3}\right)^x \cdot 9$

$45\left(\frac{1}{3}\right)^x$

$45(3)^{-x}$

Which one of the following is equivalent to $g(x) = 10(2)^{3x+1}$

A) $g(x) = 20\left(\sqrt[3]{2}\right)^x$

B) $g(x) = (20)^{3x+1}$

C) $g(x) = 20(6)^x$

D) $g(x) = 20(8)^x$

$5(2)^{3x+2}$

$5(2)^{3x}(2)^2$

$5(2)^{3x} \cdot 4$

$20(2)^{3x}$

$20(2^3)^x$

$20(8)^x$

$10(2)^{3x} \cdot 2^1$

$10(2)^{3x} \cdot 2$

$20(2)^{3x}$

$20(2^3)^x$

$20(8)^x$

Which one of the following is equivalent to $f(x) = 8(4)^{\frac{x}{2}}$

$a(b)^x$

$8(4^{\frac{1}{2}})^x$

$8(2)^x$

A) $f(x) = \frac{8}{4^x}$ $= 8\left(\frac{1}{4}\right)^x$

B) $f(x) = 16(4)^x$

C) $f(x) = (2)^{x+3}$ $= 2^x \cdot 2^3 = 2^x \cdot 8 = 8(2)^x$

D) $f(x) = 4(4)^x$

Rewrite the function in the form $a(b)^x$

$h(x) = 6(3)^{-x+1}$

$6 \cdot (3)^{-x} \cdot (3)^1$

$6 \cdot 3 \cdot (3)^{-x}$

$18(3)^{-x}$

$18(3^{-1})^x$

$18\left(\frac{1}{3}\right)^x$

Rewrite the function in the form $a(b)^x$

$$f(x) = -18\left(\frac{1}{9}\right)^{-\frac{x}{2}} = -18\left(\left(\frac{1}{9}\right)^{-\frac{1}{2}}\right)^x = -18\left(9^{\frac{1}{2}}\right)^x$$

$$= -18(3)^x$$

$$g(x) = 10(2)^{3x-2} = 10(2)^{3x}(2)^{-2}$$

$$\frac{10(2)^{3x}}{2^2}$$

$$= 10 \cdot \frac{1}{4} \cdot 2^{3x}$$

$$= \frac{5}{2} \cdot (2^3)^x$$

$$\frac{10\ (2)^{3x}}{4}$$

$$= \frac{5}{2}(8)^x$$

Let $f(x) = 3^x$

A) What sequence of transformations would map the graph of $f(x)$ to the graph of $g(x) = 3^{x+2}$

horizontal translation -2 units

B) What sequence of transformations would map the graph of $f(x)$ to the graph of $h(x) = 3^x + 2$

Vertical translation 2 units

C) What sequence of transformations would map the graph of $f(x)$ to the graph of $k(x) = 9(3)^x$

Vertical dilation by factor of 9

D) What sequence of transformations would map the graph of $f(x)$ to the graph of $m(x) = 3^{9x}$

horizontal dilation by factor of $\frac{1}{9}$

Haha! I tricked you. Super sneaky. Example A and C were the same.

$g(x) = 3^{x+2}$

$k(x) = 9(3)^x$

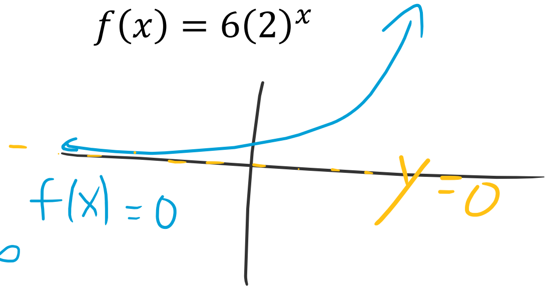
$3^x \cdot 3^2$

$3^x \cdot 9$

$9(3)^x$

Determine the end behavior of each function. Express using limit notation.

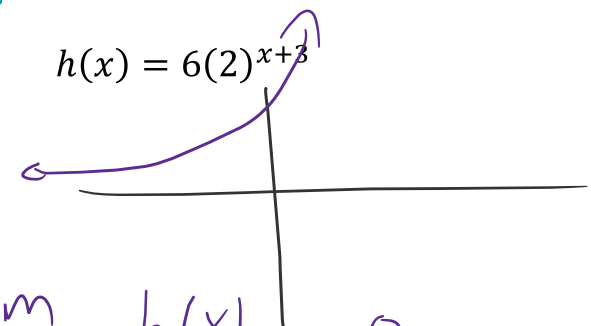
$$f(x) = 6(2)^x$$



$\lim_{x \to -\infty} f(x) = 0$

$\lim_{x \to \infty} f(x) = \infty$

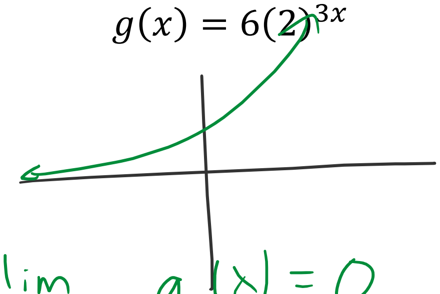
$$h(x) = 6(2)^{x+3}$$



$\lim_{x \to -\infty} h(x) = 0$

$\lim_{x \to \infty} h(x) = \infty$

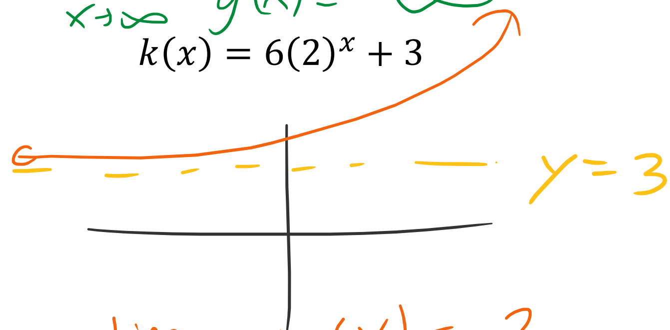
$$g(x) = 6(2)^{3x}$$



$\lim_{x \to -\infty} g(x) = 0$

$\lim_{x \to \infty} g(x) = \infty$

$$k(x) = 6(2)^x + 3$$



$\lim_{x \to -\infty} k(x) = 3$

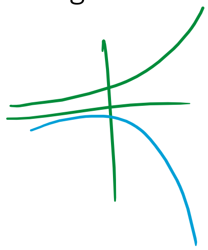
$\lim_{x \to \infty} k(x) = \infty$

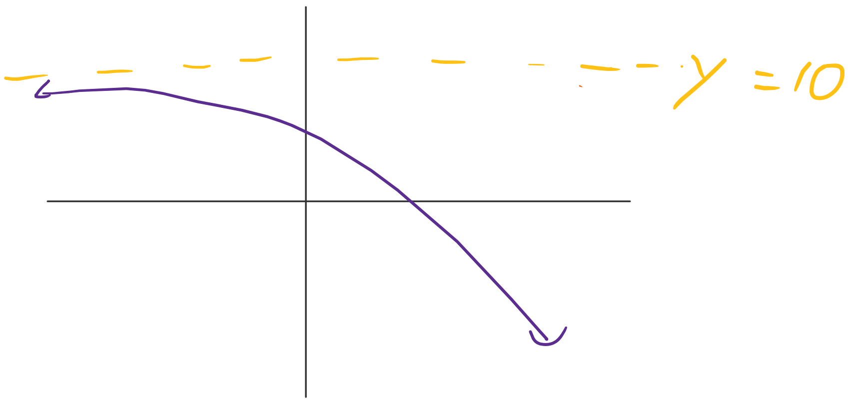
Determine the end behavior. Express using limit notation.

$r(x) = -4(2)^x + 10$

$4(2)^x$

$-4(2)^x$





$\lim_{x \to -\infty} r(x) = 10$

$\lim_{x \to \infty} r(x) = -\infty$

(Review) When we want to express end behavior with words instead of limits

Instead of $\lim_{x \to -\infty} f(x)$, we can say: "As the input values of $f(x)$ decrease without bound…"

Instead of $\lim_{x \to \infty} f(x)$, we can say: "As the input values of $f(x)$ increase without bound…"

Instead of $= \infty$, we can say: "…the output values increase without bound."

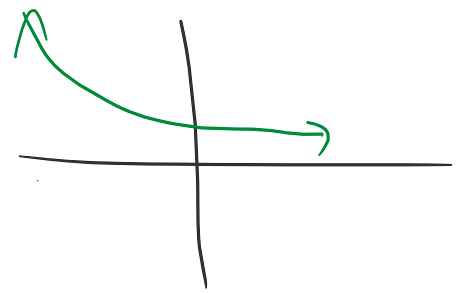
Instead of $= -\infty$, we can say: "…the output values decrease without bound."

Example: Determine the end behavior of $f(x) = -4x^3 + x^2 - 5x - 6$

(Review) When the end behavior is a horizontal asymptote

Instead of saying $\lim_{x \to -\infty} f(x) = 5$, we say "As the input values of $f(x)$ decrease without bound, the output gets arbitrarily close to 5."

Example: Determine the end behavior of $f(x) = 4\left(\frac{1}{3}\right)^x$

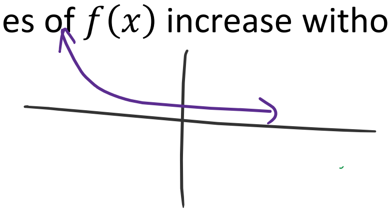


Fill in the blanks to express the end behavior of each function

$f(x) = 12(3)^{-x+5}$ $= 12(3)^{-x}\cdot(3)^5 = 12\cdot 3^5\cdot(3)^{-x} = 12\cdot 3^5\cdot\left(\frac{1}{3}\right)^x$

As the input values of $f(x)$ decrease without bound, the output values increase without bound

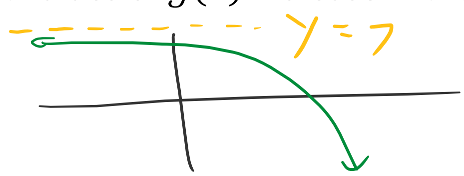
As the input values of $f(x)$ increase without bound, the output values get arbitrarily close to 0



$g(x) = -10(2)^x + 7$

As the input values of $g(x)$ decrease without bound, the output values get arbitrarily close to 7

As the input values of $g(x)$ increase without bound, the output values decrease without bound



Determine the domain and range of each of the following functions

$$f(x) = 10\left(\frac{1}{2}\right)^{-x} + 3$$

$$g(x) = -5(10)^x$$

$$h(x) = -5(3)^x + 1$$